8-7 Vectors

Use a ruler and a protractor to draw each vector. Include a scale on each diagram.
1. \( \vec{w} = 75 \text{ miles per hour} 40^\circ \text{ east of south} \)

**SOLUTION:**
Using your straight edge, make two perpendicular lines, labeling the ends N, S, E, and W, according to the correct direction. Then, place your protractor along the \( \hat{NS} \) axis and measure a 40 degree angle from the South side of the protractor toward the East axis. If you make your scale 1 inch: 60 mi/h, then your vector will be 1.2 inches.

![Diagram of vector \( \vec{w} \)](image)

1 in. : 60 mi/h

**ANSWER:**

![Diagram of vector \( \vec{w} \)](image)

1 in. : 60 mi/h
8-7 Vectors

2. \( \vec{h} = 46 \text{ feet per second} \) 170° to the horizontal

**SOLUTION:**
Using your straight edge, make two perpendicular lines, labeling the ends N, S, E, and W, according to the correct direction. Then, place your protractor along the \( \vec{WB} \) (horizontal) axis and measure a 40 degree angle from the South axis of the protractor toward the East axis.

If you make your scale 1 cm: 20 ft/s, then your vector will be \( \frac{46}{20} = 2.3 \) cm.

![Diagram of vector](image)

1 cm : 20 ft/s

**ANSWER:**

![Diagram of vector](image)

1 cm : 20 ft/s
8-7 Vectors

Copy the vectors to find each sum or difference.

\[ \overrightarrow{c} + \overrightarrow{d} \]

3.  

**SOLUTION:**

Using the Triangle Method, start by copying \( \overrightarrow{c} \) and \( \overrightarrow{d} \). Draw \( \overrightarrow{c} \), so that its tail touches the tail of \( \overrightarrow{d} \).

Draw the resultant vector from the tail of \( \overrightarrow{c} \) to the tip of \( \overrightarrow{d} \).

**ANSWER:**
4. **SOLUTION:**

Using the Triangle Method, copy \( \vec{y} \) and \( \vec{z} \). Draw \( -\vec{z} \) and translate \( \vec{y} \) so that its tip touches the tail of \( -\vec{z} \).

Draw the resulting vector from the tail of \( \vec{y} \) to the tip of \( -\vec{z} \).

**ANSWER:**
Write the component form of each vector.

5.

\[ YZ = (x_2 - x_1, y_2 - y_1) \]

\[ YZ = \left( 5 - 0, 5 - 0 \right) \]

\[ YZ = (5, 5) \]

The component form of \( YZ \) is \( (5, 5) \).

ANSWER:

\( (5, 5) \)

6.

\[ PQ = (x_2 - x_1, y_2 - y_1) \]

\[ PQ = \left( -5 - 1, -4 - (-4) \right) \]

\[ PQ = (-6, 0) \]

The component form of \( PQ \) is \( (-6, 0) \).

ANSWER:

\( (-6, 0) \)
8-7 Vectors

Find the magnitude and direction of each vector.

7. \( \vec{t} = (2, -4) \)

**SOLUTION:**

Use the Distance Formula to find the magnitude.

\[
\vec{t} = \sqrt{(2 - 0)^2 + (-4 - 0)^2} \\
\vec{t} = \sqrt{2^2 + (-4)^2} \\
\vec{t} = \sqrt{4 + 16} \\
\vec{t} = \sqrt{20}
\]

Use trigonometry to find the direction.

Graph \( \vec{t} \), its horizontal component, and its vertical component. Then, use the inverse tangent function to find \( \theta \).

\[
\tan \theta = \frac{4}{2} \\
\theta = \tan^{-1}\left(\frac{4}{2}\right) \\
\theta = 63.4
\]

Since \( \vec{t} \) is in the 4th quadrant, you can find its direction by subtracting \( \theta \) from 360. Therefore, the direction of \( \vec{t} \) is 360-63.4=296.6 degrees.

**ANSWER:**

\( \sqrt{20}; \approx 296.6 \)
8. \( \vec{f} = (-6, -5) \)

**SOLUTION:**
Use the Distance Formula to find the magnitude.

\[
\vec{f} = \sqrt{(-6 - 0)^2 + (-5 - 0)^2}
\]

\[
\vec{f} = \sqrt{(-6)^2 + (-5)^2}
\]

\[
\vec{f} = \sqrt{36 + 25}
\]

\[
\vec{f} = \sqrt{61}
\]

Use trigonometry to find the direction.

Graph \( \vec{f} \), its horizontal component, and vertical component. Then, use the inverse tangent function to find \( \theta \).

\[ \tan \theta = \frac{5}{6} \]

\[ \theta = \tan^{-1}\left(\frac{5}{6}\right) \]

\[ \theta = 39.8 \]

Since \( \vec{f} \) is in the 3rd Quadrant, add \( \theta \) to 180 to find the direction of the vector. Therefore, the direction of \( \vec{f} \) is 180 + 39.8 = 219.8 degrees.

**ANSWER:**
\[ \sqrt{61}; \approx 219.8 \]
8-7 Vectors

Find each of the following for \( \mathbf{a} = \langle -4, 1 \rangle \), \( \mathbf{b} = \langle -1, -3 \rangle \), and \( \mathbf{c} = \langle 3, 5 \rangle \).

Check your answers graphically.

9. \( \mathbf{c} + \mathbf{a} \)

SOLUTION:
You can find the sum of two vectors by adding their components.

\[
\langle 3, 5 \rangle + \langle -4, 1 \rangle = \langle 3 - 4, 5 + 1 \rangle = \langle -1, 6 \rangle
\]

ANSWER:
\( \langle -1, 6 \rangle \)
10. $2\vec{b} - \vec{a}$

**SOLUTION:**

\[
2( -1, -3) - ( -4, 1) = ( -2, -6) - ( -4, 1) \\
= ( -2 - (-4), -6 - 1) \\
= (2, -7)
\]

**ANSWER:**

$(2, -7)$

11. **CCSS MODELING** A plane is traveling due north at a speed of 350 miles per hour. If the wind is blowing from the west at a speed of 55 miles per hour, what is the resultant speed and direction that the airplane is traveling?

**SOLUTION:**

Draw a diagram. Let $\vec{r}$ represent the resultant vector.

The component form of the vector representing the plane's velocity is $(0, 350)$, and the component form of the vector representing the velocity of the wind is $(55, 0)$. 
The resultant vector is \( \langle 0, 350 \rangle + \langle 55, 0 \rangle = \langle 55, 350 \rangle \).

Use Distance formula to find the magnitude.

\[
\vec{r} = \sqrt{(55-0)^2 + (350-0)^2} \\
= \sqrt{(55)^2 + (350)^2} \\
= \sqrt{3025 + 122500} \\
= \sqrt{125525} \approx 354.3 \text{ m/h}
\]

Use trigonometry to find the resultant direction.

\[
\tan \theta = \frac{55}{350} \\
\theta = \tan^{-1} \left( \frac{55}{350} \right) \\
\theta \approx 8.9
\]

The direction of the plane 8.9 degrees East of North.

**ANSWER:**

\[ \approx 354.3 \text{ mi/h at angle of 8.9° east of north} \]
**8-7 Vectors**

*Use a ruler and a protractor to draw each vector. Include a scale on each diagram.*

12. \( \overrightarrow{g} = 60 \text{ inches per second at } 145^\circ \text{ to the horizontal} \)

**SOLUTION:**

Draw a horizontal line. Then, with your protractor on the horizontal line, measure 145 degrees. If your scale is 1 cm: 20 inches, then mark a scale that is 3 cm long (60/20 = 3).

1 cm: 20 in/s

**ANSWER:**

1 cm: 20 in/s
13. \( \vec{n} = 8 \text{ meters at an angle of } 24^\circ \text{ west of south} \)

**SOLUTION:**

Draw the NESW axes. Place your protractor on the \( \overleftrightarrow{NS} \) line and measure 24 degrees from the South axis toward the West axis. If you make your scale of 1 cm = 4 meters, then your vector will be 2 cm long (8/4=2).

1 cm: 4 m

**ANSWER:**

1 cm: 4 m
8-7 Vectors

14. \( \vec{a} = 32 \) yards per minute at 78° to the horizontal

**SOLUTION:**
Draw a horizontal line. Place your protractor on the line and measure 78 degrees. If your scale is 1 cm: 10 yards, then make your vector 3.2 cm long (32/10=3.2).

\[ \begin{array}{c}
\text{1 cm : 10 yd/min}
\end{array} \]

**ANSWER:**

\[ \begin{array}{c}
\text{1 cm : 10 yd/min}
\end{array} \]
8-7 Vectors

15. \( \vec{k} = 95 \text{ kilometers per hour at angle of } 65^\circ \text{ east of north} \)

**SOLUTION:**
Draw the NESW axes. Place your protractor on the NS axes and draw 65 degrees from the North axis toward the East axis. If your scale is 1 in: 50 km/hr, then make your vector just less than 2 inches long. (95/50=1.9)

\[ \text{1 in. : } 50 \text{ km/h} \]

**ANSWER:**

\[ \text{1 in. : } 50 \text{ km/h} \]
8-7 Vectors

Copy the vectors to find each sum or difference.

SOLUTION:
Using the Parallelogram Method of subtracting vectors, draw \( \vec{t} \) and \(-\vec{m}\) and place the tail of \(-\vec{m}\) so that it touches the tail of \(\vec{t}\). Complete the parallelogram. Then draw the diagonal.

Remove the dashed lines to reveal the resulting vector \(\vec{t} - \vec{m}\).

ANSWER:
SOLUTION:
Using the Parallelogram method, copy \( \vec{j} \) and \( -\vec{k} \) and place \( -\vec{k} \) so that its tail touches the tail of \( \vec{j} \). Complete the parallelogram and draw the diagonal.

Remove the dashed lines for the resulting vector \( \vec{j} - \vec{k} \).

ANSWER:
18.

SOLUTION:
Using the Parallelogram Method, copy \( \vec{w} \) and \( \vec{z} \) so that their tails are touching. Complete the parallelogram and draw the diagonal.

Remove the dashed lines and the resulting vector is \( \vec{w} + \vec{z} \).

ANSWER:
8-7 Vectors

\[ \vec{c} + \vec{a} \]

19.

**SOLUTION:**
Using the Parallelogram method, copy \( \vec{c} \) and \( \vec{a} \) and translate them so that their tails are touching. Complete the parallelogram and draw the diagonal.

Remove the dashed lines and the diagonal is the resulting vector \( \vec{c} + \vec{a} \).

**ANSWER:**
8-7 Vectors

SOLUTION:

Using the Parallelogram Method, copy \( \vec{d} \) and \( -\vec{f} \). Translate \( -\vec{f} \) so that its tail touches the tail of \( \vec{d} \). Complete the parallelogram and draw the diagonal.

Remove the dashed lines and the resulting vector is \( \vec{d} - \vec{f} \).

ANSWER:

SOLUTION:

Using the Parallelogram Method, copy \( \vec{t} \) and \( -\vec{m} \) and translate \( -\vec{m} \) so that its tail touches the tail of \( \vec{t} \). Complete the parallelogram and draw the diagonal.
Use a ruler and a protractor to draw each vector. Include a scale on each diagram.

1. \( \vec{v} = 75 \text{ miles per hour, 40º east of north} \)
2. \( \vec{w} = 8 \text{ meters at an angle of 24º} \)

In right triangle ABC shown below, what is the measure of \( \angle A \) to the nearest tenth of a degree?

A 32.2º

SOLUTION:

Connects the initial point of the first vector and the terminal point of the second. The resultant is the diagonal of the parallelogram.

The initial point of the resultant starts at the initial point of the first vector in both methods; however, in the triangle method, the resultant is the hypotenuse of the triangle.

\[ \begin{align*}
\vec{t} + \vec{m} &= \vec{r} \\
&= \vec{t} - \vec{m} \\
&= 2 \vec{t} - \vec{m}
\end{align*} \]

\( \vec{t} - \vec{m} \) is the resultant of Jonas speed and speed of the stream.

Jonas crosses the stream in 20 seconds and the river is 80 ft wide. Therefore, the velocity is \( \frac{80}{20} = 4 \text{ ft/sec} \), which is the resultant vector.

SOLUTION:

The direction of a vector is indicated by its terminal point. They can be on either end of the vector.

CCSS PRECISION

b. \( \vec{v} + \vec{w} \)

ANSWER:

b. \( \vec{v} - \vec{w} \)

ANSWER:

For this proof, consider substituting the values of the vectors and using vector addition and scalar multiplication:

\[ \begin{align*}
\vec{a} &= (a_1, a_2) \\
\vec{b} &= (b_1, b_2) \\
\vec{a} + \vec{b} &= (a_1 + b_1, a_2 + b_2) \\
\vec{a} - \vec{b} &= (a_1 - b_1, a_2 - b_2)
\end{align*} \]

The direction of \( \vec{a} \) is \( \theta_1 \) and the direction of \( \vec{b} \) is \( \theta_2 \). The angle \( \theta \) between the two vectors is given by

\[ \theta = \arctan \left( \frac{b_2 - a_2}{b_1 - a_1} \right) \]

Therefore, the angle \( \theta \) can be found using the inverse tangent function.

\[ \theta = \arctan \left( \frac{b_2 - a_2}{b_1 - a_1} \right) \]

SOLUTION:

The direction of \( \vec{a} \) is \( \theta_1 \) and the direction of \( \vec{b} \) is \( \theta_2 \). The angle \( \theta \) between the two vectors is given by

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Therefore, the angle \( \theta \) can be found using the inverse tangent function.

\[ \theta = \arctan \left( \frac{b_2 - a_2}{b_1 - a_1} \right) \]

SOLUTION:

We can set up a proportion to find the point \( P \):

\[ \frac{P_x}{P_y} = \frac{A_x}{A_y} \]

Therefore, the runner’s direction was 71.6 degrees east of north.

You can find the difference of two vectors by subtracting their components.

\[ \vec{u} - \vec{v} = (u_1 - v_1, u_2 - v_2) \]

SOLUTION:

You can find the difference of two vectors by subtracting their components.

\[ \vec{u} - \vec{v} = (u_1 - v_1, u_2 - v_2) \]

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SOLUTION:

You can find the difference of two vectors by subtracting their components.

\[ \vec{u} - \vec{v} = (u_1 - v_1, u_2 - v_2) \]
8-7 Vectors

Write the component form of each vector.

![Vector Diagram]

22. $\overrightarrow{WX} = (x_2 - x_1, y_2 - y_1)$

$\overrightarrow{WX} = (-1 - (-1), 5 - 1)$

$\overrightarrow{WX} = (0, 4)$

The component form of $\overrightarrow{WX}$ is $(0,4)$.

ANSWER: 
$(0,4)$

![Vector Diagram]

23. $\overrightarrow{LM} = (x_2 - x_1, y_2 - y_1)$

$\overrightarrow{LM} = (4 - (-1), 5 - 5)$

$\overrightarrow{LM} = (5, 0)$

The component form of $\overrightarrow{LM}$ is $(5,0)$.

ANSWER: 
$(5,0)$
24. **SOLUTION:**

\[ \overrightarrow{RS} = (x_2 - x_1, y_2 - y_1) \]

\[ \overrightarrow{RS} = (-3 - (-5), -1 - (-6)) \]

\[ \overrightarrow{RS} = (2, 5) \]

The component form of \( \overrightarrow{RS} \) is \( (2, 5) \).

**ANSWER:**

\( (2, 5) \)

25. **SOLUTION:**

\[ \overrightarrow{KJ} = (x_2 - x_1, y_2 - y_1) \]

\[ \overrightarrow{KJ} = (-5 - 1), -5 - (-2) \]

\[ \overrightarrow{KJ} = (-6, -3) \]

The component form of \( \overrightarrow{KJ} \) is \( (-6, -3) \).

**ANSWER:**

\( (-6, -3) \)
26. \[ \text{SOLUTION:} \]
\[
\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1)
\]
\[
\overrightarrow{AB} = \begin{pmatrix} -4 - 1 \\ 5 - 1 \end{pmatrix}
\]
\[
\overrightarrow{AB} = ( -5, 4 )
\]

The component form of \( \overrightarrow{AB} \) is \( (-5, 4) \).

\[ \text{ANSWER:} \]
\[
\langle -5, 4 \rangle
\]

27. \[ \text{SOLUTION:} \]
\[
\overrightarrow{GF} = (x_2 - x_1, y_2 - y_1)
\]
\[
\overrightarrow{GF} = \begin{pmatrix} 2 - 5 \\ -2 - 4 \end{pmatrix}
\]
\[
\overrightarrow{GF} = ( -3, -6 )
\]

The component form of \( \overrightarrow{GF} \) is \( (-3, -6) \).

\[ \text{ANSWER:} \]
\[
\langle -3, -6 \rangle
\]
28. **FIREWORKS** The ascent of a firework shell can be modeled using a vector. Write a vector in component form that can be used to describe the path of the firework shown.

![Vector Diagram]

**SOLUTION:**
\[ \vec{r} = (x_2 - x_1, y_2 - y_1) \]
\[ \vec{r} = (70 - 0, 350 - 0) \]
\[ \vec{r} = (70,350) \]

**ANSWER:**
\( (70,350) \)
8-7 Vectors

CCSS SENSE-MAKING Find the magnitude and direction of each vector.
29. \( \vec{c} = (5, 3) \)

**SOLUTION:**
Use the Distance Formula to find the magnitude.

\[
\vec{r} = \sqrt{(5 - 0)^2 + (3 - 0)^2} \\
\vec{r} = \sqrt{(5)^2 + (3)^2} \\
\vec{r} = \sqrt{25 + 9} \\
\vec{r} = \sqrt{34}
\]

Use trigonometry to find the direction.

Graph \( \vec{r} \), its horizontal component, and its vertical component. Then, use the inverse tangent function to find \( \theta \).

\[
\tan \theta = \frac{3}{5} \\
\theta = \tan^{-1}\left(\frac{3}{5}\right) \\
\theta \approx 31^\circ
\]

**ANSWER:**
\( \sqrt{34} \), \( \approx 31.0^\circ \)
8-7 Vectors

30. \( \vec{m} = (2, 9) \)

**SOLUTION:**
Use the Distance Formula to find the magnitude.

\[
\vec{r} = \sqrt{(2 - 0)^2 + (9 - 0)^2} = \sqrt{2^2 + 9^2} = \sqrt{4 + 81} = \sqrt{85}
\]

Use trigonometry to find the direction.

Graph \( \vec{r} \), its horizontal component, and its vertical component. Then, use the inverse tangent function to find \( \theta \).

\[
\tan \theta = \frac{9}{2}
\]

\[
\theta = \tan^{-1} \left( \frac{9}{2} \right)
\]

\[
\theta \approx 77.5^\circ
\]

**ANSWER:**

\( \sqrt{85}; \approx 77.5^\circ \)
31. \( \vec{z} = (-7, 1) \)

**SOLUTION:**
Use the Distance Formula to find the magnitude.

\[
\vec{r} = \sqrt{(-7 - 0)^2 + (1 - 0)^2} \\
\vec{r} = \sqrt{(-7)^2 + 1^2} \\
\vec{r} = \sqrt{49 + 1} \\
\vec{r} = \sqrt{50}
\]

Use trigonometry to find the direction.

Graph \( \vec{r} \), its horizontal component, and its vertical component. Then, use the inverse tangent function to find \( \theta \).

\[
\tan \theta = \frac{1}{7} \\
\theta = \tan^{-1} \left( \frac{1}{7} \right) \\
\theta \approx 8.1^\circ
\]

Since \( \theta \) is in the 2nd quadrant, we can find the direction by subtracting it from 180 degrees.

\[
180 - 8.1 = 171.9 \text{ degrees}
\]

**ANSWER:**

\( \sqrt{50}; \approx 171.9^\circ \)
32. $\vec{d} = (4, -8)$

**SOLUTION:**
Use the Distance Formula to find the magnitude.

\[
\vec{r} = \sqrt{(4 - 0)^2 + (-8 - 0)^2} \\
\vec{r} = \sqrt{(4)^2 + (-8)^2} \\
\vec{r} = \sqrt{16 + 64} \\
\vec{r} = \sqrt{80}
\]

Use trigonometry to find the direction.

Graph $\vec{r}$, its horizontal component, and its vertical component. Then, use the inverse tangent function to find $\theta$.

\[
\tan \theta = \frac{8}{4} \\
\theta = \tan^{-1}\left(\frac{8}{4}\right) \\
\theta \approx 63.4
\]

Since $\theta$ is in the 4th Quadrant, we can subtract it from 360 to get the direction of the vector.

\[
360 - 63.4 = 296.6 \text{ degrees}
\]

**ANSWER:**
\[
\sqrt{80}; \approx 296.6^\circ
\]
8-7 Vectors

33. \( \vec{k} = (-3, -6) \)

**SOLUTION:**

Use the Distance Formula to find the magnitude.

\[
\vec{r} = \sqrt{(-3 - 0)^2 + (-6 - 0)^2}
\]

\[
\vec{r} = \sqrt{(-3)^2 + (-6)^2}
\]

\[
\vec{r} = \sqrt{9 + 36}
\]

\[
\vec{r} = \sqrt{45}
\]

Use trigonometry to find the direction.

Graph \( \vec{r} \), its horizontal component, and its vertical component. Then, use the inverse tangent function to find \( \theta \).

\[
\tan \theta = \frac{6}{3}
\]

\[
\theta = \tan^{-1} \left( \frac{6}{3} \right)
\]

\[
\theta \approx 63.4°
\]

Since \( \theta \) is in the 3rd Quadrant, we can add it to 180 degrees to find the direction of the vector.

\[
180 + 63.4 = 243.4°
\]

**ANSWER:**

\[
\sqrt{45}; \approx 243.4°
\]
34. \( \vec{q} = (-9, -4) \)

**SOLUTION:**
Use the Distance Formula to find the magnitude.

\[
\vec{r} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \\
\vec{r} = \sqrt{(-9-0)^2 + (-4-0)^2} \\
\vec{r} = \sqrt{81 + 16} \\
\vec{r} = \sqrt{97}
\]

Use trigonometry to find the direction.

Graph \( \vec{r} \), its horizontal component, and its vertical component. Then, use the inverse tangent function to find \( \theta \).

\[
\tan \theta = \frac{4}{9} \\
\theta = \tan^{-1}\left(\frac{4}{9}\right) \\
\theta \approx 24.0
\]

Since \( \theta \) is in the 3rd Quadrant, we can add it to 180 degrees to find the direction of the vector.

\[180 + 24.0 = 204.0 \text{ degrees}\]

**ANSWER:**

\[\sqrt{97}; \approx 204.0^\circ\]
8-7 Vectors

Find each of the following for \( \vec{a} = (-3, -5), \vec{b} = (2, 4), \) and \( \vec{c} = (3, -1). \)

Check your answers graphically.

35. \( \vec{b} + \vec{c} \)

**SOLUTION:**
You can find the sum of two vectors by adding their components.

\[
(2, 4) + (3, -1) = (2 + 3, 4 + (-1)) = (5, 3)
\]

**ANSWER:**
\((5, 3)\)
36. $\vec{c} + \vec{a}$

**SOLUTION:**
You can find the sum of two vectors by adding their components.

$$\langle 3, -1 \rangle + \langle -3, -5 \rangle = \langle 3 + (-3), -1 + (-5) \rangle = \langle 0, -6 \rangle$$

**ANSWER:**
$$\langle 0, -6 \rangle$$
37. $\vec{b} - \vec{c}$

**SOLUTION:**
You can find the difference of two vectors by subtracting their components.

$\langle 2, 4 \rangle - \langle 3, -1 \rangle = \langle 2 - 3, 4 - (-1) \rangle = \langle -1, 5 \rangle$

**ANSWER:**
$\langle -1, 5 \rangle$
38. \( \vec{a} - \vec{c} \)

**SOLUTION:**
You can find the difference of two vectors by subtracting their components.

\[
\langle -3, -5 \rangle - \langle 3, -1 \rangle = \langle -3 - 3, -5 - (-1) \rangle = \langle -6, -4 \rangle
\]

**ANSWER:**
\( \langle -6, -4 \rangle \)
39. $2\vec{c} - \vec{a}$

**SOLUTION:**
You can multiply a vector by a scale factor of 2 by multiplying each component by 2. You can find the difference of two vectors by subtracting their components.

$$2(3, -1) - (-3, -5) = (6, -2) - (-3, -5)$$
$$= (6 - (-3), -2 - (-5))$$
$$= (9, 3)$$

**ANSWER:**
$(9, 3)$
40. \(2\vec{b} + \vec{c}\)

**SOLUTION:**
To find 2 times a vector, you can multiply each component by a scalar factor of 2. To find the sum of two vectors, you can add the components.

\[
2(2, 4) + (3, -1) = (4, 8) + (3, -1) \\
= (4 + 3, 8 + (-1)) \\
= (7, 7)
\]
41. HIKING Amy hiked due east for 2 miles and then hiked due south for 3 miles.
   a. Draw a diagram to represent the situation, where \( \vec{r} \) is the resultant vector.
   b. How far and in what direction is Amy from her starting position?

**SOLUTION:**
You can find the distance traveled by using the Pythagorean Theorem.

\[
2^2 + 3^2 = \vec{r}^2
\]
\[
4 + 9 = \vec{r}^2
\]
\[
13 = \vec{r}^2
\]
\[
\vec{r} = \sqrt{13} \approx 3.6 \text{ miles}
\]

Use trigonometry to find the direction of Amy's hike.

\[
\tan \theta = \frac{3}{2}
\]
\[
\theta = \tan^{-1}\left(\frac{3}{2}\right)
\]
\[
\theta = 56.3^\circ
\]

Since direction is measured from the north-south line, we can subtract \( \theta \) from 90 degrees.

\[
90 - 56.3 = 36.7
\]

Amy is traveling at a 36.7 degree angle east of south.

**ANSWER:**
   a.

\[
\]

b. 3.6 mi at an angle of 33.7° east of south
42. **EXERCISE** A runner’s velocity is 6 miles per hour due east, with the wind blowing 2 miles per hour due north.
   a. Draw a diagram to represent the situation, where \( \vec{r} \) is the resultant vector.
   b. What is the resultant velocity of the runner?

**SOLUTION:**

You can find the speed traveled by using the Pythagorean Theorem.

\[
2^2 + 6^2 = r^2
\]

\[
4 + 36 = r^2
\]

\[
40 = r^2
\]

\[
r = \sqrt{40} \approx 6.3 \text{ mi/hr}
\]

You can use trigonometry to determine the direction traveled.

\[
\tan \theta = \frac{2}{6}
\]

\[
\theta = \tan^{-1}\left(\frac{2}{6}\right)
\]

\[
\theta = 18.4 ^\circ
\]

We express direction as it relates to the north-south line.

\[
90 - 18.4 = 71.6
\]

Therefore, the runner’s direction was 71.6 degrees east of north.

**ANSWER:**

a. 

b. 6.3 mi/h at an angle of 71.6° east of north
8-7 Vectors

Find each of the following for \( \vec{f} = (-4, -2), \vec{g} = (6, 1), \) and \( \vec{h} = (2, -3). \)

43. \( \vec{f} + \vec{g} + \vec{h} \)

\[ \begin{align*}
\langle -4, -2 \rangle + \langle 6, 1 \rangle + \langle 2, -3 \rangle &= \langle -4 + 6 + 2, -2 + 1 - 3 \rangle \\
&= \langle 4, -4 \rangle
\end{align*} \]

**SOLUTION:**
\( \langle 4, -4 \rangle \)

**ANSWER:**
\( \langle 4, -4 \rangle \)

44. \( \vec{h} - 2\vec{f} + \vec{g} \)

\[ \begin{align*}
\langle 2, -3 \rangle - 2\langle -4, -2 \rangle + \langle 6, 1 \rangle &= \langle 2, -3 \rangle - \langle -8, -4 \rangle + \langle 6, 1 \rangle \\
&= \langle 2 + 8, -3 - 4 + 1 \rangle \\
&= \langle 16, 2 \rangle
\end{align*} \]

**SOLUTION:**
\( \langle 16, 2 \rangle \)

**ANSWER:**
\( \langle 16, 2 \rangle \)

45. \( 2\vec{g} - 3\vec{f} + \vec{h} \)

\[ \begin{align*}
2\langle 6, 1 \rangle - 3\langle -4, -2 \rangle + \langle 2, -3 \rangle &= \langle 12, 2 \rangle - \langle -12, -6 \rangle + \langle 2, -3 \rangle \\
&= \langle 12 + 12 + 2, 2 + 6 + (-3) \rangle \\
&= \langle 26, 5 \rangle
\end{align*} \]

**SOLUTION:**
\( \langle 26, 5 \rangle \)

**ANSWER:**
\( \langle 26, 5 \rangle \)

46. **HOMECOMING** Nikki is on a committee to help plan her school’s homecoming parade. The parade starts at the high school and continues as shown in the figure at the right.

a. What is the magnitude and direction of the vector formed from the ending point of the parade to the school?

b. Find the sum of the two vectors to determine the length of the parade if 1 unit = 0.25 mile.
8-7 Vectors

a) The vector formed by the starting point at the school and the terminal point at the end of the parade is \((5, -1)\). Use the distance formula to find the magnitude of the vector.

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(5 - 0)^2 + ((-1) - 0)^2}
\]

\[
= \sqrt{(5)^2 + (-1)^2}
\]

\[
= \sqrt{26} \text{ or about 5.1}
\]

Use trigonometry to find the direction of the vector.

\[
\tan \theta = \frac{1}{5}
\]

\[
\theta = \tan^{-1}\left(\frac{1}{5}\right)
\]

\[
\theta = 11.3
\]

The parade started at the origin and terminated below the positive x-axis in quadrant IV. The direction of the vector is the angle that it makes clockwise from the positive x-axis, which is 360° − 11.3° or 348.7°.

Therefore, the magnitude of the vector is about 5.1 units and the direction is at an angle of about 348.7°.

b) Use the distance formula to find the length of each leg of the parade.

First, find the length of the leg between (5, 2) and (0, 0).

\[
d_1 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(5 - 0)^2 + (2 - 0)^2}
\]

\[
= \sqrt{(5)^2 + (2)^2}
\]

\[
= \sqrt{29} \text{ or about 5.4}
\]

Next, find the length of the leg between (5, -1) and (5, 2).

\[
d_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

\[
= \sqrt{(5 - 5)^2 + (2 - (-1))^2}
\]

\[
= \sqrt{(3)^2}
\]

\[
= 3
\]

So, the total distance of the parade is about 5.4 + 3 or 8.4 units.

Use the conversion rate of 1 unit = 0.25 mile to determine the length of the parade in miles.
8-7 Vectors

\[ 1 \text{ unit} = 0.25 \text{ mile} \]
\[ 8.4 \text{ units} = 0.25(8.4) \text{ miles} \]
\[ = 2.1 \text{ miles} \]

The length of the parade is about 2.1 miles.

**ANSWER:**

a. \( \sqrt{26} \approx 5.1 \text{ units}, \ 348.7^\circ \)
b. 2.1 mi

47. **SWIMMING** Jonas is swimming from the east bank to the west bank of a stream at a speed of 3.3 feet per second. The stream is 80 feet wide and flows south. If Jonas crosses the stream in 20 seconds, what is the speed of the current?

**SOLUTION:**
Jonas crosses the stream in 20 seconds and the river is 80 ft wide. Therefore, the velocity is 80/20 or 4 ft/sec, which is the resultant of Jonas speed and speed of the stream.

Use Pythagorean Theorem to solve for \( x \), the speed of the stream.

\[ x^2 + 3.3^2 = 4^2 \]
\[ x^2 + 10.89 = 16 \]
\[ x^2 = 5.11 \]
\[ x = \sqrt{5.11} \approx 2.3 \]

The speed of the current is about 2.3 ft/sec.

**ANSWER:**

2.3 ft/s

48. **CHALLENGE** Find the coordinates of point \( P \) on \( AB \) that partitions the segment into the given ratio \( AP \) to \( PB \).

a. A(0, 0), B(0, 6), 2 to 1
b. A(0, 0), B(-15, 0), 2 to 3

**SOLUTION:**
a.
We can set up a proportion to find the point \( P \) which divides \( AB \) into a 2:1 ratio.

\[
\frac{AP}{PB} = \frac{x}{6-x}
\]

\[
\frac{2}{1} = \frac{x}{6-x}
\]

\[
2(6 - x) = x
\]

\[
12 - 2x = x
\]

\[
12 = 3x
\]

\[
x = 4
\]

Therefore, the length of \( AP \)=4 and \( P \) is located at (4,0).

b. We can set up a proportion to find the point \( P \) which divides \( AB \) into a 2:3 ratio.

\[
\frac{AP}{PB} = \frac{x}{15-x}
\]

\[
\frac{2}{3} = \frac{x}{15-x}
\]

\[
2(15 - x) = 3x
\]

\[
30 - 2x = 3x
\]

\[
30 = 5x
\]

\[
x = 6
\]

Therefore, the length of \( AP \)=6 and \( P \) is located at (-6,0).

**ANSWER:**
8-7 Vectors

a. (0, 4)
b. (−6, 0)

49. **CCSS PRECISION** Are parallel vectors *sometimes, always, or never* opposite vectors? Explain.

**SOLUTION:**
The direction of a vector is indicated by its terminal point. They can be on either end of the vector.

Sample answer: Sometimes; Parallel vectors either have the same or opposite directions.

**ANSWER:**
Sample answer: Sometimes; Parallel vectors either have the same or opposite directions.
8-7 Vectors

PROOF  Prove each vector property. Let \( \vec{a} = (x_1, y_1) \) and \( \vec{b} = (x_2, y_2) \).

50. commutative: \( \vec{a} + \vec{b} = \vec{b} + \vec{a} \)

SOLUTION:
If \( \vec{a} = (x_1, y_1) \) and \( \vec{b} = (x_2, y_2) \), then we can substitute these values in the equation.

\[
\vec{a} + \vec{b} = (x_1, y_1) + (x_2, y_2) \quad \text{Given}
\]

\[
\vec{a} + \vec{b} = (x_1 + x_2, y_1 + y_2) \quad \text{Vector Addition}
\]

\[
\vec{a} + \vec{b} = (x_2 + x_1, y_2 + y_1) \quad \text{Commutative property of +}
\]

\[
\vec{a} + \vec{b} = (x_2, y_2) + (x_1, y_1) \quad \text{Vector Addition}
\]

\[
\vec{a} + \vec{b} = \vec{b} + \vec{a} \quad \text{Substitution}
\]

ANSWER:
\[
\vec{a} + \vec{b} = (x_1, y_1) + (x_2, y_2) \quad \text{Given}
\]

\[
\vec{a} + \vec{b} = (x_1 + x_2, y_1 + y_2) \quad \text{Vector Addition}
\]

\[
\vec{a} + \vec{b} = (x_2 + x_1, y_2 + y_1) \quad \text{Commutative property of +}
\]

\[
\vec{a} + \vec{b} = (x_2, y_2) + (x_1, y_1) \quad \text{Vector Addition}
\]

\[
\vec{a} + \vec{b} = \vec{b} + \vec{a} \quad \text{Substitution}
\]
8-7 Vectors

51. scalar multiplication: $k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$, where $k$ is a scalar

**SOLUTION:**
For this proof, consider substituting the values of the vectors and using vector addition and scalar multiplication properties on the vectors in component form.

\[
k\left( \vec{a} + \vec{b} \right) = k((x_1, y_1) + (x_2, y_2)) \quad \text{Substitution}
\]
\[
k\left( \vec{a} + \vec{b} \right) = k\left((x_1 + x_2, y_1 + y_2)\right) \quad \text{Vector Addition}
\]
\[
k\left( \vec{a} + \vec{b} \right) = (k(x_1 + x_2), k(y_1 + y_2)) \quad \text{Scalar Multiplication}
\]
\[
k\left( \vec{a} + \vec{b} \right) = \left\{ kx_1 + kx_2, ky_1 + ky_2 \right\} \quad \text{Scalar Multiplication}
\]
\[
k\left( \vec{a} + \vec{b} \right) = (kx_1, ky_1) + (kx_2, ky_2) \quad \text{Vector Addition}
\]
\[
k\left( \vec{a} + \vec{b} \right) = k(x_1, y_1) + k(x_2, y_2) \quad \text{Scalar Multiplication}
\]
\[
k\left( \vec{a} + \vec{b} \right) = k\vec{a} + k\vec{b} \quad \text{Substitution}
\]

**ANSWER:**

\[
k\left( \vec{a} + \vec{b} \right) = k((x_1, y_1) + (x_2, y_2)) \quad \text{Substitution}
\]
\[
k\left( \vec{a} + \vec{b} \right) = k\left((x_1 + x_2, y_1 + y_2)\right) \quad \text{Vector Addition}
\]
\[
k\left( \vec{a} + \vec{b} \right) = (k(x_1 + x_2), k(y_1 + y_2)) \quad \text{Scalar Multiplication}
\]
\[
k\left( \vec{a} + \vec{b} \right) = \left\{ kx_1 + kx_2, ky_1 + ky_2 \right\} \quad \text{Scalar Multiplication}
\]
\[
k\left( \vec{a} + \vec{b} \right) = (kx_1, ky_1) + (kx_2, ky_2) \quad \text{Vector Addition}
\]
\[
k\left( \vec{a} + \vec{b} \right) = k(x_1, y_1) + k(x_2, y_2) \quad \text{Scalar Multiplication}
\]
\[
k\left( \vec{a} + \vec{b} \right) = k\vec{a} + k\vec{b} \quad \text{Substitution}
\]
8-7 Vectors

52. OPEN ENDED Draw a pair of parallel vectors.
   a. Draw the sum of the two vectors. What is true of the direction of the vector representing the sum?
   b. Draw the difference of the two vectors. What is true of the vector representing the difference?

   SOLUTION:
   a. When drawing two vectors to add, you need to consider the direction both tips must be heading.

   ![Vector Sum Diagram]

   Sample answer: The sum of the two vectors is also parallel to both of the original vectors.

   b. When drawing two vectors to subtract, you need to consider the direction each tip must be heading.

   ![Vector Difference Diagram]

   Sample answer: The difference of the two vectors is also parallel to both of the original vectors.

   ANSWER:
   a. 

   ![Vector Sum Diagram]

   Sample answer: The sum of the two vectors is also parallel to both of the original vectors.

   b. 

   ![Vector Difference Diagram]

   Sample answer: The difference of the two vectors is also parallel to both of the original vectors.
53. **WRITING IN MATH** Compare and contrast the parallelogram and triangle methods of adding vectors.

**SOLUTION:**
The initial point of the resultant starts at the initial point of the first vector in both methods; however, in the parallelogram method, both vectors start at the same initial point, whereas in the triangle method, the resultant connects the initial point of the first vector and the terminal point of the second. The resultant is the diagonal of the parallelogram in the parallelogram method.

**Parallelogram method: tails touch**

![Parallelogram method: tails touch](image)

**Triangle method: tail touches tip**

![Triangle method: tail touches tip](image)

**ANSWER:**
The initial point of the resultant starts at the initial point of the first vector in both theorems; however, in the parallelogram method, both vectors start at the same initial point, whereas in the triangle method, the resultant connects the initial point of the first vector and the terminal point of the second. The resultant is the diagonal of the parallelogram in the parallelogram method.
54. **EXTENDED RESPONSE** Sydney parked her car and hiked along two paths described by the vectors \( \langle 2,3 \rangle \) and \( \langle 5,-1 \rangle \).

a. What vector represents her hike along both paths?

b. When she got to the end of the second path, how far is she from her car if the numbers represent miles?

**SOLUTION:**

a) Find the resultant vector by adding both vectors.
\[
\langle 2,3 \rangle + \langle 5,-1 \rangle = \langle 2 + 5, 3 + (-1) \rangle = \langle 7, 2 \rangle
\]

b) Find the magnitude of the resultant vector.
\[
v = \sqrt{x^2 + y^2}
\]
\[
= \sqrt{7^2 + 2^2}
\]
\[
= \sqrt{49 + 4}
\]
\[
= \sqrt{53}
\]
\[
\approx 7.3
\]

Sydney is 7.3 miles apart from her car.

**ANSWER:**

a. \( \langle 2,3 \rangle + \langle 5,-1 \rangle = \langle 7,2 \rangle \)

b. \( \sqrt{7^2 + 2^2} = \sqrt{53} \) or 7.3 mi
8-7 Vectors

55. In right triangle $ABC$ shown below, what is the measure of $\angle A$ to the nearest tenth of a degree?

\[ \sin A = \frac{\text{opposite}}{\text{hypotenuse}} \]

\[ A = \frac{22}{35} \approx 0.6286 \]

\[ A = \sin^{-1}(0.6286) \approx 38.9 \]

The correct choice is B.

ANSWER: B

56. PROBABILITY A die is rolled. Find the probability of rolling a number greater than 4 to the nearest hundredth.

F 0.17
G 0.33
H 0.50
J 0.67

SOLUTION:
Probability = \( \frac{\text{possible outcomes}}{\text{total outcomes}} \)

The total number of outcomes is 6.
The possible number of outcomes is 2, the numbers 5 and 6.

\[ \text{Probability} = \frac{2}{6} \approx 0.33 \]

Therefore, the correct choice is G.

ANSWER: G

57. SAT/ACT Caleb followed the two paths shown below to get to his house $C$ from a store $S$. What is the total
distance of the two paths, in meters, from C to S?

A 10.8 m
B 24.5 m
C 31.8 m
D 35.3 m
E 38.4 m

**SOLUTION:**

Use the Pythagorean theorem to find the length of each path.

First, find the length of Path 1.

For that right triangle, the length of the legs are, (28 – 4)m or 24m and (15 – 10)m or 5m.

\[
24^2 + 5^2 = (Path 1)^2
\]

\[
576 + 25 = (Path 1)^2
\]

\[
601 = (Path 1)^2
\]

\[
Path 1 = \sqrt{601} \approx 24.5
\]

Now, find the length of Path 2.

\[
4^2 + 10^2 = (Path 2)^2
\]

\[
16 + 100 = (Path 2)^2
\]

\[
116 = (Path 2)^2
\]

\[
Path 2 = \sqrt{116} \approx 10.8
\]

Add both the distances to find the total distance.

\[
CS = 24.5 + 10.8 = 35.3 m
\]

Therefore, the correct choice is D.

**ANSWER:**

D
Find \( x \). Round angle measures to the nearest degree and side measures to the nearest tenth.

\[ \text{SOLUTION:} \]

We are given the measures of two angles and a nonincluded side, so use the Law of Sines to write a proportion.

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin 21^\circ}{23} = \frac{\sin 128^\circ}{x} \]

\[ x \sin 21^\circ = 23 \sin 128^\circ \]

\[ x = \frac{23 \sin 128^\circ}{\sin 21^\circ} \approx 50.6 \]

\[ \text{ANSWER:} \]

50.6

\[ \text{SOLUTION:} \]

We are given the measures of two angles and a nonincluded side, so use the Law of Sines to write a proportion.

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin x^\circ}{68} = \frac{\sin 48^\circ}{53} \]

\[ 53 \sin x^\circ = 68 \sin 48^\circ \]

\[ \sin x^\circ = \frac{68 \sin 48^\circ}{53} \approx 0.95 \]

\[ x^\circ = \sin^{-1}(0.95) \approx 72 \]

\[ \text{ANSWER:} \]

72.0
8-7 Vectors

SOLUTION:
We are given the measures of two angles and a nonincluded side, so use the Law of Sines to write a proportion.

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin 111^\circ}{x}
\]

\[
\frac{41}{x} = \frac{\sin 111^\circ}{x}
\]

\[
x \sin 111^\circ = 41 \sin 111^\circ
\]

\[
x = \frac{41 \sin 111^\circ}{\sin 19^\circ}
\]

\[
\approx 117.6
\]

ANSWER:
117.6

61. SOCCER Adelina is in a soccer stadium 80 feet above the field. The angle of depression to the field is 12°. What is the horizontal distance between Adelina and the soccer field?

SOLUTION:
Draw a figure.

A is the position of Adelina and C is the position of the soccer field.

Use the tangent ratio to find the value of x.

\[
\tan C = \frac{\text{opposite}}{\text{adjacent}}
\]

\[
\tan 12^\circ = \frac{80}{x}
\]

\[
x \tan 12^\circ = 80
\]

\[
x = \frac{80}{\tan 12^\circ}
\]

\[
\approx 376.4
\]

The horizontal distance between Adelina and the soccer field is 376.4 ft.

ANSWER:
376.4 ft
**8-7 Vectors**

**Quadrilateral WXYZ is a rectangle. Find each measure if \( m \angle 1 = 30 \).**

![Quadrilateral WXYZ diagram]

62. \( m \angle 2 \)

**SOLUTION:**

All four angles of a rectangle are right angles. So,

\[
\begin{align*}
\angle 1 + \angle 2 &= 90 \\
\angle 2 &= 90 - 30 \\
&= 60
\end{align*}
\]

**ANSWER:**

60

63. \( m \angle 8 \)

**SOLUTION:**

Since the diagonals of a rectangle are congruent and bisect each other, the triangle with the angles 1, 11 and 8 is an isosceles triangle.

So, \( m \angle 1 = m \angle 8 \) and \( m \angle 1 = m \angle 8 = 30 \).

**ANSWER:**

30

64. \( m \angle 12 \)

**SOLUTION:**

Since the diagonals of a rectangle are congruent and bisect each other, the triangle with the angles 1, 11 and 8 is an isosceles triangle.

\[
\begin{align*}
m \angle 1 &= m \angle 8 \text{ and } m \angle 1 = m \angle 8 = 30.
\end{align*}
\]

The sum of the three angles in a triangle is 180.

\[
\begin{align*}
m \angle 1 + m \angle 8 + \angle 11 &= 180 \\
m \angle 11 &= 180 - 30 - 30 \\
&= 120
\end{align*}
\]

**ANSWER:**

120
8-7 Vectors

65. \( m \angle 5 \)

**SOLUTION:**
The measures of the angles 1 and 5 are equal as they are alternate interior angles.

\[ m \angle 1 = m \angle 5 = 30 \]

**ANSWER:**
30

66. \( m \angle 6 \)

**SOLUTION:**
Since this is a rectangle, all of the vertices form right angles. Therefore,

\[ m \angle 1 + m \angle 2 = 90 \]
\[ 30 + m \angle 2 = 90 \]
\[ m \angle 2 = 90 - 30 \]
\[ m \angle 2 = 60 \]

The measures of the angles 2 and 6 are equal as they are alternate interior angles.

\[ m \angle 2 = m \angle 6 = 60 \]

**ANSWER:**
60

67. \( m \angle 3 \)

**SOLUTION:**
Since this is a rectangle, all of the vertices form right angles. Therefore,

\[ m \angle 1 + m \angle 2 = 90 \]
\[ 30 + m \angle 2 = 90 \]
\[ m \angle 2 = 90 - 30 \]
\[ m \angle 2 = 60 \]

Since the diagonals of a rectangle are congruent and bisect each other, the triangle with the angles 2, 10 and 3 is an isosceles triangle.

So, \( m \angle 2 = m \angle 3 \) and \( m \angle 2 = m \angle 3 = 60 \)

**ANSWER:**
60 degrees
8-7 Vectors

Assume that segments and angles that appear to be congruent in each figure are congruent. Indicate which triangles are congruent.

68. **SOLUTION:**
Use the symmetry of the bridge to find congruent corresponding sides between triangles.

\[ \Delta 1 \equiv \Delta 10, \Delta 2 \equiv \Delta 9, \]
\[ \Delta 3 \equiv \Delta 8, \Delta 4 \equiv \Delta 7, \]
\[ \Delta 5 \equiv \Delta 6 \]

**ANSWER:**
\[ \Delta 1 \equiv \Delta 10, \Delta 2 \equiv \Delta 9, \]
\[ \Delta 3 \equiv \Delta 8, \Delta 4 \equiv \Delta 7, \]
\[ \Delta 5 \equiv \Delta 6 \]

69. **SOLUTION:**
Use the symmetry of the design to find congruent corresponding angles and sides in the triangles.

\[ \Delta s 1 - 4, \Delta s 5 - 12, \]
\[ \Delta s 13 - 20 \]

**ANSWER:**
\[ \Delta s 1 - 4, \Delta s 5 - 12, \]
\[ \Delta s 13 - 20 \]
70.

**SOLUTION:**
Assuming that the innermost figure is a square, we know it is composed of 4 congruent isosceles right triangles. Additionally, the triangles that surround the square share the same longer leg (side of the square). Continue to look for congruent triangles as they spiral out from the center of the square.

\[\Delta s 1, 5, 6, \text{ and } 11.\]
\[\Delta s 3, 8, 10, \text{ and } 12.\]
\[\Delta s 2, 4, 7, \text{ and } 9.\]

**ANSWER:**
\[\Delta s 1, 5, 6, \text{ and } 11, \Delta s 3, 8, 10,\]
and \[12, \Delta s 2, 4, 7, \text{ and } 9\]