10-2 Measuring Angles and Arcs

Find the value of $x$.

1.

**SOLUTION:**
The sum of the measures of the central angles of a circle with no interior points in common is 360.

\[
\begin{align*}
60 + 130 + x &= 360 & \text{Sum of Central Angles} \\
190 + x &= 360 & \text{Simplify} \\
x &= 360 - 190 & \text{Subtract 190 from each side} \\
x &= 170 & \text{Simplify.}
\end{align*}
\]

**ANSWER:** 170

2.

**SOLUTION:**
The sum of the measures of the central angles of a circle with no interior points in common is 360.

\[
\begin{align*}
140 + 35 + 35 + x &= 360 & \text{Sum of Central Angles} \\
210 + x &= 360 & \text{Simplify} \\
x &= 360 - 210 & \text{Subtract 210 from each side} \\
x &= 150 & \text{Simplify.}
\end{align*}
\]

**ANSWER:** 150

---

CCSS PRECISION $\overline{HK}$ and $\overline{IG}$ are diameters of $O\ LaTeX{L}. Identify each arc as a major arc, minor arc, or semicircle. Then find its measure.

3. $\overline{IJ}$

**SOLUTION:**
Here, $\overline{IJ}$ is the longest arc connecting the points $I$ and $J$ on $O\ LaTeX{L}$. Therefore, it is a major arc.

$\overline{IJ}$ is a major arc that shares the same endpoints as minor arc $IJ$.

\[
m(\text{arc}IJ) = 360 - m(\text{arc}IJ) \quad \text{Measure of Major Arc Rule}
\]

\[
= 360 - 90 \quad \text{Substitution}
\]

\[
= 270 \quad \text{Simplify}
\]

**ANSWER:** major arc; 270

4. $\overline{HI}$

**SOLUTION:**
Here, $\overline{HI}$ is the shortest arc connecting the points $I$ and $H$ on $O\ LaTeX{L}$. Therefore, it is a minor arc.

\[
m(\text{arc}HI) = m\angle HI \quad \text{Measure of Minor Arc Rule}
\]

\[
= 59 \quad \text{Substitution}
\]

**ANSWER:** minor arc; 59
5. \( \overline{HGK} \)

**SOLUTION:**
Here, \( \overline{HK} \) is a diameter. Therefore, \( \overline{HGK} \) is a semicircle.
The measure of a semicircle is 180, so \( m\overline{HGK} = 180 \).

**ANSWER:**
semicircle; 180

6. **RESTAURANTS** The graph shows the results of a survey taken by diners relating what is most important about the restaurants where they eat.

![Graph showing restaurant preferences](image)

**a.** Find \( m\overline{AB} \).

**b.** Find \( m\overline{BC} \).

**c.** Describe the type of arc that the category Great Food represents.

**SOLUTION:**
**a.** Here, \( \overline{AB} \) is a minor arc.
The measure of the arc is equal to the measure of the central angle. Find the 22% of 360 to find the central angle.
\[
m(\text{arc } AB) = 0.22(360) = 79.2 \quad \text{Find 22% of 360 Simplify.}
\]

**b.** Here, \( \overline{BC} \) is a minor arc.
The measure of the arc is equal to the measure of the central angle. Find the 8% of 360 to find the central angle.
\[
m(\text{arc } BC) = 0.08(360) = 28.8 \quad \text{Find 8% of 360 Simplify.}
\]

**c.** The arc that represents the category Great Food \( \overline{CD} \), is the longest arc connecting the points \( C \) and \( D \). Therefore, it is a major arc.

**ANSWER:**
- a. 79.2
- b. 28.8
- c. major arc
Find the value of $x$.

1. **SOLUTION:**

The sum of the measures of the central angles of a circle with no interior points in common is 360. The measure of an arc equals 360 minus the measure of the minor arc corresponding central angle. The measure of a major arc equals the measure of the minor arc equals the measure of the central angle.

Thus, find the arc measure.

Find the length of $\overline{JK}$. Round to the nearest hundredth.

10. **SOLUTION:**

Use the arc length equation with $r = KC$ or 2 and $x$.

$\ell = \frac{m\angle K}{2\pi} \cdot 2\pi$  
Arc Length Equation

$\ell = \frac{30}{360} \cdot 2\pi$  
Substitution

Therefore, the length of $\overline{JK}$ is about 1.05 feet.

ANSWER:

1.05 ft

11. **SOLUTION:**

The diameter of $\odot C$ is 15 centimeters, so the radius is 7.5 centimeters. The $m\angle KJC = m\angle KCJ$ or 105.

Use the equation to find the arc length.

$\ell = \frac{X}{360} \cdot 2\pi$  
Arc Length Equation

$\ell = \frac{105}{360} \cdot 2\pi(7.5)$  
Substitution

$\approx 13.74$  
Use a calculator

Therefore, the length of $\overline{JK}$ is about 13.74 centimeters.

ANSWER:

13.74 cm
10-2 Measuring Angles and Arcs

Find the value of $x$.

12. $\text{SOLUTION:}$

The sum of the measures of the central angles of a circle with no interior points in common is 360.

\[125 + 155 + x = 360\]
\[280 + x = 360\]
\[x = 360 - 280\]
\[x = 80\]

\text{ANSWER:} 80

13. $\text{SOLUTION:}$

The sum of the measures of the central angles of a circle with no interior points in common is 360.

\[65 + 70 + x = 360\]
\[135 + x = 360\]
\[x = 360 - 135\]
\[x = 225\]

\text{ANSWER:} 225

14. $\text{SOLUTION:}$

The sum of the measures of the central angles of a circle with no interior points in common is 360.

\[150 + 85 + 90 + x = 360\]
\[325 + x = 360\]
\[x = 360 - 325\]
\[x = 35\]

\text{ANSWER:} 35

15. $\text{SOLUTION:}$

The sum of the measures of the central angles of a circle with no interior points in common is 360.

\[135 + 145 + x + x = 360\]
\[280 + 2x = 360\]
\[2x = 80\]
\[x = 40\]

\text{ANSWER:} 40
10-2 Measuring Angles and Arcs

\( \overline{AD} \) and \( \overline{CG} \) are diameters of \( \odot B \). Identify each arc as a major arc, minor arc, or semicircle. Then find its measure.

16. \( m\overline{CD} \)

**SOLUTION:**
Here, \( \overline{CD} \) is the shortest arc connecting the points \( C \) and \( D \) on \( \odot B \). Therefore, it is a minor arc.

\[
m(\text{arc} \overline{CD}) = m \angle CBD \\
= 55 \quad \text{Measure of Minor Arc Rule} \\
\]

**ANSWER:**
minor arc; 55

17. \( m\overline{AC} \)

**SOLUTION:**
Here, \( \overline{AC} \) is the shortest arc connecting the points \( A \) and \( C \) on \( \odot B \). Therefore, it is a minor arc.

Since \( \overline{AD} \) is a diameter, arc \( \overline{ACD} \) is a semicircle and has a measure of 180. Use angle addition to find the measure of arc \( \overline{AC} \).

\[
m(\text{arc} \overline{AC}) = m(\text{arc} \overline{ACD}) + m(\text{arc} \overline{CD}) \\
= 180 - 55 \quad \text{Subtract 55 from each side} \\
= 125 \quad \text{Arc Addition Postulate} \\
\]

Therefore, the measure of arc \( \overline{AC} \) is 125.

**ANSWER:**
minor arc; 125

18. \( m(\text{arc} \overline{CFG}) \)

**SOLUTION:**
Here, \( \overline{CG} \) is a diameter. Therefore, arc \( \overline{CFG} \) is a semicircle and \( m(\text{arc} \overline{CFG}) = 180 \).

**ANSWER:**
semicircle; 180

19. \( m\overline{CGD} \)

**SOLUTION:**
Here, \( \overline{CGD} \) is the longest arc connecting the points \( C \) and \( D \) on \( \odot B \). Therefore, it is a major arc.

Arc \( \overline{CGD} \) is a major arc that shares the same endpoints as minor arc \( \overline{CD} \).

\[
m(\text{arc} \overline{CGD}) = 360 - m(\text{arc} \overline{CD}) \quad \text{Measure of Major Arc Rule} \\
= 360 - 55 \quad m(\text{arc} \overline{CD}) = m \angle CBD \\
= 305 \quad \text{Simplify} \\
\]

Therefore, the measure of arc \( \overline{CGD} \) is 305.

**ANSWER:**
major arc; 305

20. \( m\overline{GCF} \)

**SOLUTION:**
Here, \( \overline{GCF} \) is the longest arc connecting the points \( G \) and \( F \) on \( \odot B \). Therefore, it is a major arc.

Arc \( \overline{GCF} \) shares the same endpoints as minor arc \( \overline{GF} \).

\[
m(\text{arc} \overline{GCF}) = 360 - m(\text{arc} \overline{GF}) \quad \text{Measure of Major Arc Rule} \\
= 360 - 35 \quad m(\text{arc} \overline{GF}) = m \angle GFB \\
= 325 \quad \text{Simplify} \\
\]

Therefore, the measure of arc \( \overline{GCF} \) is 325.

**ANSWER:**
major arc; 325

21. \( m\overline{ACD} \)

**SOLUTION:**
Here, \( \overline{AD} \) is a diameter. Therefore, \( \overline{ACD} \) is a semicircle.

The measure of a semicircle is 180, so \( m(\text{arc} \overline{ACD}) = 180 \).

**ANSWER:**
semicircle; 180
22. $m\overline{AG}$

**SOLUTION:**
Here, $\overline{AG}$ is the shortest arc connecting the points $A$ and $G$ on $\overline{OB}$. Therefore, it is a minor arc.
The measure of a minor arc is equal to the measure of its related central angle.

$m\angle AFG = m\angle CBD$ (Vertical angles are congruent)

$m\angle AFG = 55$ (Substitution)

$m(\text{arc } AG) = m\angle AFG$ (Measure of Minor Arc Rule)

$m(\text{arc } AG) = 55$ (Substitution)

Therefore, the measure of arc $AG$ is 55.

**ANSWER:**
minor arc; 55

23. $m\overline{ACF}$

**SOLUTION:**
Here, $\overline{ACF}$ is the longest arc connecting the points $A$ and $F$ on $\overline{OB}$. Therefore, it is a major arc.
Major arc $ACF$ shares the same endpoints as minor arc $AF$, so $m(\text{arc } ACF) = 360 - m(\text{arc } AF)$.
Since $\angle AFG$ and $\angle CBD$ are vertical angles,

$m\angle AFG = m\angle CBD$ or 55.

$m(\text{arc } ACF) = 360 - m(\text{arc } AF)$ (Substitution)

$m(\text{arc } ACF) = 360 - 55$

$m(\text{arc } ACF) = 270$ (Simplify)

Therefore, the measure of arc $ACF$ is 270.

**ANSWER:**
major arc; 270

24. **SHOPPING** The graph shows the results of a survey in which teens were asked where the best place was to shop for clothes.

a. What would be the arc measures associated with the mall and vintage stores categories?

b. Describe the kinds of arcs associated with the category “Mall” and category “None of these”.

c. Are there any congruent arcs in this graph? Explain.

**SOLUTION:**

a. The measure of the arc is equal to the measure of the central angle. The mall contributes 76% and the vintage stores contribute 4% of the total shopping. Find the 76% of 360 to find the central angle of the arc associated with the malls.

\[
\frac{76}{100} \times 360 = 273.6
\]

Find the 4% of 360 to find the central angle of the arc associated with the vintage stores.

\[
\frac{4}{100} \times 360 = 14.4
\]

b. The arc associated with the mall has a measure of 273.6. So, it is a major arc. The arc associated with none of these has a measure of 9% of 360 or 32.4. So, it is a minor arc.

c. Yes; the arcs associated with the online and none of these categories have the same arc measure since each category accounts for the same percentage of the circle, 9%.

**ANSWER:**
a. 273.6, 14.4
b. major arc; minor arc

c. Yes; the arcs associated with the online and none of these categories have the same arc measure since each category accounts for the same percentage of the circle, 9%.
25. **CCSS MODELING** The table shows the results of a survey in which Americans were asked how long food could be on the floor and still be safe to eat.

a. If you were to construct a circle graph of this information, what would be the arc measures associated with the first two categories?
b. Describe the kind of arcs associated with the first category and the last category.
c. Are there any congruent arcs in this graph? Explain.

### Dropped Food

<table>
<thead>
<tr>
<th>Do you eat food dropped on the floor?</th>
<th>Not safe to eat</th>
<th>78%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-second rule*</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>Five-second rule*</td>
<td>8%</td>
<td></td>
</tr>
<tr>
<td>Ten-second rule*</td>
<td>4%</td>
<td></td>
</tr>
</tbody>
</table>

* The length of time the food is on the floor.

**SOLUTION:**

a. The measure of the arc is equal to the measure of the central angle. The “not so safe” category contributes 78% and the “three-second rule” contributes 10% in the supporters in the survey. Find the 78% of 360 to find the central angle of the arc associated with the not so safe category.

\[0.78 \times 360 = 280.8\]

Find the 10% of 360 to find the central angle of the arc associated with the three-second rule category.

\[0.10 \times 360 = 36\]

b. The arc corresponding to not safe to eat category measures 280.8, so it is a major arc. Similarly, the arc corresponding to the ten-second rule measures 4% of 360 or 14.4, so it is a minor arc.

c. No; no categories share the same percentage of the circle.

**ANSWER:**
a. 280.8; 36
b. major arc; minor arc
c. No; no categories share the same percentage of the circle.

### Entertainment

**ENTERTAINMENT** Use the Ferris wheel shown to find each measure.

26. \( \widehat{mFG} \)

**SOLUTION:**

The measure of the arc is equal to the measure of the central angle. We have, \( m\angle FLG = 40 \).

Therefore, \( m\widehat{FG} = 40 \).

**ANSWER:**

40

27. \( \widehat{mJH} \)

**SOLUTION:**

The measure of the arc is equal to the measure of the central angle. We have, \( m\angle JLH = 60 \).

Therefore, \( m\widehat{JH} = 60 \).

**ANSWER:**

60

28. \( \widehat{mJKF} \)

**SOLUTION:**

Here, \( JF \) is a diameter. Therefore, \( \widehat{JKF} \) is a semicircle and \( m\widehat{JKF} = 180 \).

**ANSWER:**

180
29. \( m \overarc{JFH} \)

**SOLUTION:**
Arc \( JFH \) is a major arc.

\[
m(\overarc{JFH}) = 360 - m(\overarc{JH}) \quad \text{Measure of Major Arc Rule} \\
= 360 - 60 \quad m(\overarc{JFH}) = m \angle JFH \\
= 300 \quad \text{Simplify.}
\]

Therefore, the measure of arc \( JFH \) is 300.

**ANSWER:**
300

30. \( m \overarc{GHF} \)

**SOLUTION:**
Arc \( GHF \) is a major arc.

\[
m(\overarc{GHF}) = 360 - m(\overarc{GP}) \quad \text{Measure of Major Arc Rule} \\
= 360 - 40 \quad m(\overarc{GHF}) = m \angle GPF \\
= 320 \quad \text{Simplify.}
\]

Therefore, the measure of arc \( GHF \) is 320.

**ANSWER:**
320

31. \( m \overarc{GHK} \)

**SOLUTION:**
Here, \( GK \) is a diameter. Therefore, \( GHK \) is a semicircle and \( m \overarc{GHK} = 180 \).

**ANSWER:**
180

32. \( m \overarc{HK} \)

**SOLUTION:**
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. First find \( m \angle JLK \) and then use the Arc Addition Postulate.

\[
m \angle JLK = m \angle FLG \quad \text{Vertical angles are congruent}
\]

\[
m \angle JLK = 40 \quad \text{Substitution} \\
m(\overarc{JHK}) = m(\overarc{JHE}) + m(\overarc{KHE}) = 40 + 100 \quad \text{Simple}
\]

Therefore, the measure of arc \( HK \) is 100.

**ANSWER:**
100

33. \( m \overarc{JKG} \)

**SOLUTION:**
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. \( JFG \) is a diameter, so arc \( JF \) is a semicircle and has a measure of 180.

\[
m(\overarc{JKG}) = m(\overarc{JF}) + m(\overarc{FG}) \quad \text{Arc Addition Postulate} \\
= 180 + 40 \quad m(\overarc{FG}) = m \angle FGO \\
= 220 \quad \text{Simplify.}
\]

Therefore, the measure of arc \( JKG \) is 220.

**ANSWER:**
220

34. \( m \overarc{KFH} \)

**SOLUTION:**
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. \( KG \) and \( FJ \) are diameters, so arc \( KG \) and arc \( FJ \) are semicircles with measures of 180. Find \( m \angle GLH \) and then use the Arc Addition Postulate.

\[
m(\overarc{KFH}) = m(\overarc{KG}) + m(\overarc{FG}) \quad \text{Arc Addition Postulate} \\
= 180 + 40 \quad m(\overarc{FG}) = m \angle FGO \\
= 220 \quad \text{Simplify.}
\]

Therefore, the measure of arc \( KFH \) is 260.

**ANSWER:**
260

35. \( m \overarc{HGF} \)

**SOLUTION:**
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. Use semicircle \( FJ \) to find \( m \angle HLG \) and then use the Arc Addition Postulate.

\[
m(\overarc{HGF}) = m(\overarc{FG}) + m(\overarc{OC}) \quad \text{Arc Addition Postulate} \\
= 180 + 40 \quad m(\overarc{OC}) = m \angle OCG \\
= 220 \quad \text{Simplify.}
\]

Therefore, the measure of arc \( HGF \) is 120.

**ANSWER:**
120
10-2 Measuring Angles and Arcs

Use \( \bigcirc P \) to find the length of each arc. Round to the nearest hundredth.

![Diagram](image)

36. \( \widehat{RS} \) if the radius is 2 inches

**SOLUTION:**

Use the arc length equation with \( r = 2 \) inches and \( m(\text{arc } RS) = m \angle RPS \) or 130.

\[
\ell = \frac{x}{360} \cdot 2\pi
\]

\[
= \frac{130}{360} \cdot 2\pi(2)
\]

\[
\approx 4.54 \text{ inches}
\]

Therefore, the length of arc RS is about 4.54 inches.

**ANSWER:**

4.54 in.

37. \( \widehat{QT} \), if the diameter is 9 centimeters

**SOLUTION:**

Use the arc length equation with \( r = \frac{1}{2}(9) \) or 4.5 centimeters and \( m(\text{arc } QT) = m \angle QPT \) or 112.

\[
\ell = \frac{x}{360} \cdot 2\pi
\]

\[
= \frac{112}{360} \cdot 2\pi(4.5)
\]

\[
\approx 8.80 \text{ cm}
\]

Therefore, the length of arc QT is about 8.80 centimeters.

**ANSWER:**

8.80 cm

38. \( QR \) if \( PS = 4 \) millimeters

**SOLUTION:**

\( \overline{PS} \) is a radius of \( \bigcirc P \) and \( \overline{RT} \) is a diameter. Use the Arc Addition Postulate to find \( m(\text{arc } QR) \).

\[
m(\text{arc } RT) = m(\text{arc } QR) + m(\text{arc } QT)
\]

\[
180 = m(\text{arc } QR) + 112
\]

\[
68 = m(\text{arc } QR)
\]

Use the arc length equation with \( r = 4 \) millimeters and \( x = m(\text{arc } RQ) \) or 68.

\[
\ell = \frac{x}{360} \cdot 2\pi
\]

\[
= \frac{68}{360} \cdot 2\pi(4)
\]

\[
\approx 4.75 \text{ mm}
\]

Therefore, the length of arc QR is about 4.75 millimeters.

**ANSWER:**

4.75 mm

39. \( RS \) if \( RT = 15 \) inches

**SOLUTION:**

\( \overline{RT} \) is a diameter, so \( r = \frac{1}{2}(15) \) or 7.5 inches. Use the arc length equation with \( x = m(\text{arc } RS) \) or 130.

\[
\ell = \frac{x}{360} \cdot 2\pi
\]

\[
= \frac{130}{360} \cdot 2\pi(7.5)
\]

\[
\approx 17.02 \text{ in.}
\]

Therefore, the length of arc RS is about 17.02 inches.

**ANSWER:**

17.02 in.
40. \( \overline{QRS} \) if \( RT = 11 \) feet

**SOLUTION:**

Use the Arc Addition Postulate to find the \( m(\text{arc } QR) \).

\[
\begin{align*}
180 &= m(\text{arc } QR) + m(\text{arc } QT) & \text{Arc Addition Postulate} \\
180 &= m(\text{arc } QR) + 112 & m(\text{arc } QT) = m(\text{arc } QT) \\
68 &= m(\text{arc } QR) & \text{Simplify}
\end{align*}
\]

Use the Arc Addition Postulate to find \( m(\text{arc } QRS) \).

\[
\begin{align*}
m(\text{arc } QRS) &= m(\text{arc } QR) + m(\text{arc } RS) & \text{Arc Addition Postulate} \\
&= 68 + 130 & m(\text{arc } RS) = m(\text{arc } RS) \\
&= 198 & \text{Simplify}
\end{align*}
\]

Use the arc length equation with \( r = \frac{1}{2}(RT) \) or 5.5 feet and \( x = m(\text{arc } QRS) \) or 198.

\[
\begin{align*}
\ell &= \frac{x}{360} \cdot 2\pi & \text{Arc Length Equation} \\
&= \frac{198}{360} \cdot 2\pi(5.5) & \text{Substitution} \\
&\approx 19.01 & \text{Use a calculator}
\end{align*}
\]

Therefore, the length of arc \( QRS \) is about 19.01 feet.

**ANSWER:**

19.01 ft

41. \( \overline{RTS} \) if \( PQ = 3 \) meters

**SOLUTION:**

Arc \( RTS \) is a major arc that shares the same endpoints as minor arc \( KS \).

\[
\begin{align*}
m(\text{arc } RTS) &= 360 - m(\text{arc } RS) & \text{Measure of Major Arc Rule} \\
&= 360 - 110 & m(\text{arc } RS) = m(\text{arc } RS) \\
&= 230 & \text{Simplify}
\end{align*}
\]

Use the arc length equation with \( r = PS \) or 3 meters and \( x = m(\text{arc } RTS) \) or 230.

\[
\begin{align*}
\ell &= \frac{x}{360} \cdot 2\pi & \text{Arc Length Equation} \\
&= \frac{230}{360} \cdot 2\pi(3) & \text{Substitution} \\
&\approx 12.04 & \text{Use a calculator}
\end{align*}
\]

Therefore, the length of arc \( RTS \) is about 12.04 meters.

**ANSWER:**

12.04 m

HISTORY The figure shows the stars in the Betsy Ross flag referenced at the beginning of the lesson.

42. What is the measure of central angle \( A \)? Explain how you determined your answer.

**SOLUTION:**

There are 13 stars arranged in a circular way equidistant from each other. So, the measure of the central angle of the arc joining any two consecutive stars will be equal to \( \frac{360}{13} \approx 27.7 \).

**ANSWER:**

\( \frac{360}{13} \) stars \( \approx 27.7 \) between each star.
43. If the diameter of the circle were doubled, what would be the effect on the arc length from the center of one star \( B \) to the next star \( C \)?

**SOLUTION:**
The measure of the arc between any two stars is about 27.7. Let \( \ell_1 \) be the arc length of the original circle and \( \ell_2 \) be the arc length for the circle when the diameter is doubled. Use the arc length equation with a radius of \( r \) and \( x = 27.7 \) to find \( \ell_1 \).

\[
\ell = \frac{x}{360} \cdot 2\pi r
\]

Substitution

\[
\ell_1 = \frac{27.7}{360} \cdot 2\pi r
\]

Simplify.

Use the arc length equation with a radius of \( 2r \) and \( x = 27.7 \) to find \( \ell_2 \).

\[
\ell = \frac{x}{360} \cdot 2\pi r
\]

Substitution

\[
\ell_2 = \frac{27.7}{360} \cdot 2\pi (2r)
\]

Simplify.

The arc length for the second circle is twice the arc length for the first circle. Therefore, if the diameter of the circle is doubled, the arc length from the center of star \( B \) to the center of the next star \( C \) would double.

**ANSWER:**
The length of the arc would double.

44. **FARMS** The *Pizza Farm* in Madera, California, is a circle divided into eight equal slices, as shown at the right. Each “slice” is used for growing or grazing pizza ingredients.

a. What is the total arc measure of the slices containing olives, tomatoes, and peppers?

b. The circle is 125 feet in diameter. What is the arc length of one slice? Round to the nearest hundredth.

**SOLUTION:**
a. The circle is divided into eight equal slices. So, the measure of the central angle of each slice will be \( \frac{360}{8} = 45 \). Therefore, total arc measure of the slices containing olives, tomatoes, and peppers will be \( 3(45) = 135 \).

b. The length of an arc \( l \) is given by the formula,

\[
l = \frac{x}{360} \cdot 2\pi r
\]

where \( x \) is the central angle of the arc \( l \) and \( r \) is the radius of the circle.

The measure of the central angle of each slice will be \( \frac{360}{8} = 45 \). So, \( x = 45 \) and \( r = 62.5 \) ft. Then, arc length of each slice is \( \frac{45}{360} \cdot 2\pi (62.5) \approx 49.09 \) ft.

**ANSWER:**
a. 135
b. 49.09 ft
CCSS REASONING Find each measure. Round each linear measure to the nearest hundredth and each arc measure to the nearest degree.

45. circumference of \( \odot S \)

\[ \text{SOLUTION:} \]

The circumference of circle \( S \) is given by \( 2\pi r \). Use the arc length equation and solve for the value of \( 2\pi r \).

\[ t = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation} \]

\[ 7.94 = \frac{70}{360} \cdot 2\pi r \quad \text{Substitution} \]

\[ 40.83 = 2\pi r \quad \text{Multiply each side by} \ \frac{360}{70} \]

Therefore, the circumference of circle \( S \) is about 40.83 inches.

**ANSWER:** 40.83 in.

46. \( \overarc{CD} \)

\[ 1.31 \text{ m} \]

\[ 0.5 \text{ m} \]

\[ \text{SOLUTION:} \]

The radius of circle \( B \) is 0.5 meters and the length of arc \( CD \) is 1.31 meters. Use the arc length equation and solve for \( x \) to find the measure of arc \( CD \).

\[ t = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation} \]

\[ 1.31 = \frac{x}{360} \cdot 2\pi (0.5) \quad \text{Substitution} \]

\[ 1.31 = \frac{x \pi}{360} \quad \text{Simplify} \]

\[ 150 \approx x \quad \text{Multiply each side by} \ \frac{360}{\pi} \]

Therefore, the measure of arc \( CD \) is about 150°.

**ANSWER:** 150°

47. radius of \( \odot K \)

\[ J \quad L \quad K \]

\[ \odot K \]

\[ 56.37 \text{ ft} \]

\[ \text{SOLUTION:} \]

The measure of major arc \( JL \) is 340 and its arc length is 56.37 feet. Use the arc length equation to solve for the radius of circle \( K \).

\[ t = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation} \]

\[ 56.37 = \frac{340}{360} \cdot 2\pi \quad \text{Substitution} \]

\[ 56.37 = \frac{680\pi}{360} \quad \text{Simplify} \]

\[ 9.50 \approx r \quad \text{Multiply each side by} \ \frac{360}{680\pi} \]

Therefore, the radius of circle \( K \) is about 9.50 feet.

**ANSWER:** 9.50 ft

48. \( \overarc{EF} \)

\[ \text{SOLUTION:} \]

Here, \( \angle HCG \) and \( \angle HCD \) form a linear pair. So, the sum of their measures is 180.

\[ m\angle HCG + m\angle HCD = 180 \quad \text{Definition of Linear Pair} \]

\[ (2x) + (5x + 28) = 180 \quad \text{Substitution} \]

\[ 3x = 152 \quad \text{Subtract 28 from each side} \]

\[ x = 51 \quad \text{Divide each side by 3} \]

So, \( m\angle HCG = 2(19) \) or 38 and \( m\angle HCD = 6(19) + 28 \) or 142.

\( \overline{HE} \) is a diameter of circle \( C \), so arc \( HFE \) is a semicircle and has a measure of 180.

\[ m\angle HFE = 180 \quad \text{Definition of Semicircle} \]

\[ m\angle HFE = 180 \quad \text{Substitution} \]

Therefore, the measure of arc \( EF \) is 52.

**ANSWER:** 52
51. **RIDES** A pirate ship ride follows a semi-circular path, as shown in the diagram.

a. What is \( \widehat{AB} \)?

b. If \( CD = 62 \) feet, what is the length of \( \widehat{AB} \)? Round to the nearest hundredth.

\[
\text{SOLUTION:}
\]

**a.** From the figure, \( \widehat{AB} \) is \( 22 + 22 = 44^\circ \) less than the semi-circle centered at \( C \). Therefore,

\[
m\widehat{AB} = 180 - 44 = 136.
\]

**b.** The length of an arc \( l \) is given by the formula,

\[
l = \frac{x}{360} \cdot 2\pi r
\]

where \( x \) is the central angle of the arc \( l \) and \( r \) is the radius of the circle.

Here, \( x = 136 \) and \( r = 62 \). Use the formula.

\[
\text{length of } \widehat{AB} = \frac{136}{360} (2\pi \cdot 62)
\]

\[
\approx 147.17 \text{ ft}
\]

**ANSWER:**

a. 136

b. 147.17 ft
10-2 Measuring Angles and Arcs

52. PROOF Write a two-column proof of Theorem 10.1.

Given: \( \angle BAC \cong \angle DAE \)

Prove: \( BC \cong DE \)

**SOLUTION:**

**Proof:**

Statements (Reasons)
1. \( \angle BAC \cong \angle DAE \) (Given)
2. \( m\angle BAC = m\angle DAE \) (Definition of \( \cong \) s)
3. \( m\angle BAC = m\overline{BC} \), \( m\angle DAE = m\overline{DE} \) (Definition of arc measure)
4. \( m\overline{BC} = m\overline{DE} \) (Substitution)
5. \( \overline{BC} \cong \overline{DE} \) (Definition of \( \cong \) arcs)

**ANSWER:**

Proof:

Statements (Reasons)
1. \( \angle BAC \cong \angle DAE \) (Given)
2. \( m\angle BAC = m\angle DAE \) (Definition of \( \cong \) s)
3. \( m\angle BAC = m\overline{BC} \), \( m\angle DAE = m\overline{DE} \) (Definition of arc measure)
4. \( m\overline{BC} = m\overline{DE} \) (Substitution)
5. \( \overline{BC} \cong \overline{DE} \) (Definition of \( \cong \ arcs)

53. COORDINATE GEOMETRY In the graph, point \( M \) is located at the origin. Find each measure in \( \odot M \). Round each linear measure to the nearest hundredth and each arc measure to the nearest tenth degree.

**a.** \( m\overline{JL} \)

**b.** \( m\overline{KL} \)

**c.** \( m\overline{JK} \)

**d.** length of \( \overline{JL} \)

**e.** length of \( \overline{JK} \)

**SOLUTION:**

**a.** The measure of arc \( JL \) equals the measure of the related central angle \( JML \). Construct a right triangle by drawing \( MJ \) and a perpendicular segment from \( J \) to the \( x \)-axis. The legs of the right triangle will have lengths of 5 and 12.

Use a trigonometric ratio to find the angle of the triangle at \( M \) which equals the measure of central angle \( JML \).

\[
\tan \theta = \frac{opp}{adj} \quad \text{Definition of Tangent}
\]

\[
\tan(\angle JML) = \frac{12}{5} \quad \text{Substitution}
\]

\[
m\angle JML = \tan^{-1}\left(\frac{12}{5}\right) \quad \text{Use the inverse tangent ratio}
\]

\[
m\angle JML \approx 67.4 \quad \text{Use a calculator.}
\]

Therefore, \( m\angle JML \approx 67.4 \).

**b.** The measure of arc \( KL \) equals the measure of the related central angle \( KML \). Construct a right triangle by drawing \( MK \) and a perpendicular segment from \( K \) to the \( x \)-axis. The legs of the right triangle will have lengths of 12 and 5.

Use a trigonometric ratio to find the angle of the triangle at \( M \) which equals the measure of central angle \( KML \).

\[
\tan \theta = \frac{opp}{adj} \quad \text{Definition of Tangent}
\]

\[
\tan(\angle KML) = \frac{5}{12} \quad \text{Substitution}
\]

\[
m\angle KML = \tan^{-1}\left(\frac{5}{12}\right) \quad \text{Use the inverse tangent ratio}
\]

\[
m\angle KML \approx 22.6 \quad \text{Use a calculator.}
\]

Therefore, \( m\angle KML \approx 22.6 \).

**c.** Use the Arc Addition Postulate to find \( m(\text{arc} \ JK) \).

\[
m(\text{arc} \ JL) = m(\text{arc} \ JE) + m(\text{arc} \ EL) \quad \text{Arc Addition Postulate}
\]

\[
67.4 = m(\text{arc} \ JE) + 44.8 \quad \text{Substitute}
\]

\[
44.8 = m(\text{arc} \ EL) \quad \text{Subtract 22.6 from each side}
\]

So, the measure of arc \( JK \) is 44.8.

**d.** Use the right triangle from part \( a \) and find the length of \( MJ \) which is a radius of circle \( M \).

\[
c^2 = a^2 + b^2 \quad \text{Pythagorean Theorem}
\]

\[
MJ^2 = 5^2 + 12^2 \quad \text{Substitution}
\]

\[
MJ = 13 \quad \text{Simplify.}
\]

\[
MJ = 13 \quad \text{Take the positive square root of each side}
\]

Use the arc length equation with \( r = 13 \) and \( x = m(\text{arc} \ JL) \) or 67.4.

\[
\ell = \frac{x}{360} \cdot 2\pi r \quad \text{Arc Length Equation}
\]

\[
= \frac{67.4}{360} \cdot 2\pi(13) \quad \text{Substitution}
\]

\[
\approx 15.29 \quad \text{Use a calculator.}
\]
Therefore, the length of arc $JL$ is about 15.29 units.

e. Use the arc length equation with $r = 13$ and $x = m$ (arc $JK$) or 44.8.

$$\ell = \frac{x}{360} \cdot 2\pi r$$

Arc Length Equation

$$= \frac{44.8}{360} \cdot 2\pi (13)$$

Substitution

$$\approx 10.16$$

Use a calculator

Therefore, the length of arc $JK$ is about 10.16 units.

**ANSWER:**

a. 67.4
b. 22.6
c. 44.8
d. 15.29 units
e. 10.16 units

54.

**ARC LENGTH AND Radian MEASURE** In this problem, you will use concentric circles to show that the length of the arc intercepted by a central angle of a circle is dependent on the circle’s radius.

a. Compare the measures of arc $\ell_1$ and arc $\ell_2$.

Then compare the lengths of arc $\ell_1$ and arc $\ell_2$.

What do these two comparisons suggest?

b. Use similarity transformations (dilations) to explain why the length of an arc $\ell$ intercepted by a central angle of a circle is proportional to the circle’s radius $r$. That is, explain why we can say that for this diagram, $\frac{\ell_1}{r_1} = \frac{\ell_2}{r_2}$.

c. Write expressions for the lengths of arcs $\ell_1$ and $\ell_2$. Use these expressions to identify the constant of proportionality $k$ in $\ell = kr$.

d. The expression that you wrote for $k$ in part c gives the radian measure of an angle. Use it to find the radian measure of an angle measuring 90°.

**SOLUTION:**

a. By definition, the measure of an arc is equal to the measure of its related central angle. So, $m(\text{arc } \ell_1) = x$ and $m(\text{arc } \ell_2) = x$. By the transitive property, $m(\text{arc } \ell_1) = m(\text{arc } \ell_2)$. Using the arc length equation, $\ell_1 = \frac{x}{360}(2\pi r_1)$ and $\ell_2 = \frac{x}{360}(2\pi r_2)$. Since $r_1 < r_2$, then $\ell_1 < \ell_2$.

These comparisons suggest that arc measure is not affected by the size of the circle, but arc length is affected.

b. Since all circles are similar, the larger circle is a dilation of the smaller by some factor $k$, so $r_2 = kr_1$ or $k = \frac{r_2}{r_1}$. Likewise, the arc intercepted on the larger circle is a dilation of the arc intercepted on the smaller circle, so $\ell_2 = k\ell_1$, or $k = \frac{\ell_2}{\ell_1}$. Then substitute for $k$.

$$\frac{r_2}{r_1} = \frac{\ell_2}{\ell_1}$$

Substitute

$$r_2 \ell_1 = r_1 \ell_2$$ Cross products are equal

$$\frac{\ell_1}{r_1} = \frac{\ell_2}{r_2}$$ Divide each side by $r_1r_2$.

c. $\ell_1 = \frac{x}{360}(2\pi r_1)$ or $\frac{\pi x}{180} (r_1)$ and $\ell_2 = \frac{x}{360}(2\pi r_2)$ or $\frac{\pi x}{180} (r_2)$; Therefore, for $\ell = kr$, $k = \frac{\pi}{180}$.

d. For an angle measuring 90°, $x = 90$. Then, $k = \frac{90\pi}{180}$ or $\frac{\pi}{2}$.

**ANSWER:**

a. $m(\text{arc } \ell_1) = m(\text{arc } \ell_2)$; $\ell_1 < \ell_2$; these comparisons suggest that arc measure is not affected by the size of the circle, but arc length is affected.

b. Since all circles are similar, the larger circle is a dilation of the smaller by some factor $k$, so $r_2 = kr_1$ or $k = \frac{r_2}{r_1}$. Likewise, the arc intercepted on the larger circle is a dilation of the arc intercepted on the smaller circle, so $\ell_2 = k\ell_1$, or $k = \frac{\ell_2}{\ell_1}$. Thus $\frac{r_2}{r_1} = \frac{\ell_2}{\ell_1}$ or $\frac{\ell_1}{r_1} = \frac{\ell_2}{r_2}$.

c. $\ell_1 = \frac{\pi r_1 x}{180}$ and $\ell_2 = \frac{\pi r_2 x}{180}$; $k = \frac{\pi}{180}$.

d. $\frac{\pi}{2}$
10-2 Measuring Angles and Arcs

55. ERROR ANALYSIS Brody says that $\overrightarrow{WX}$ and $\overrightarrow{YZ}$ are congruent since their central angles have the same measure. Selena says they are not congruent. Is either of them correct? Explain your reasoning.

**SOLUTION:**
Brody has incorrectly applied Theorem 10.1. The arcs are congruent if and only if their central angles are congruent and the arcs and angles are in the same circle or congruent circles. The circles containing arc $WX$ and arc $YZ$ are not congruent because they do not have congruent radii. The arcs will have the same degree measure but will have different arc lengths. So, the arcs are not congruent. Therefore, Selena is correct.

**ANSWER:**
Selena; the circles are not congruent because they do not have congruent radii. So, the arcs are not congruent.

**CCSS ARGUMENTS** Determine whether each statement is sometimes, always, or never true. Explain your reasoning.

56. The measure of a minor arc is less than 180.

**SOLUTION:**
By definition, an arc that measures less than 180 is a minor arc. Therefore, the statement is always true.

**ANSWER:**
Always; by definition, an arc that measures less than 180 is a minor arc.

57. If a central angle is obtuse, its corresponding arc is a major arc.

**SOLUTION:**
Obtuse angles intersect arcs between 90° and 180°. So, the corresponding arc will measure less than 180°. Therefore, the statement is never true.

**ANSWER:**
Never; obtuse angles intersect arcs between 90° and 180°.

58. The sum of the measures of adjacent arcs of a circle depends on the measure of the radius.

**SOLUTION:**
Postulate 10.1 says that the measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs. The measure of each arc would equal the measure of its related central angle. The radius of the circle does not depend on the radius of the circle. Therefore, the statement is never true.

**ANSWER:**
Never; the sum of the measures of adjacent arcs depends on the measures of the arcs.

59. CHALLENGE The measures of $\overrightarrow{LM}$, $\overrightarrow{MN}$ and $\overrightarrow{NL}$ are in the ratio 5:3:4. Find the measure of each arc.

**SOLUTION:**
If the measures of arc $LM$, arc $MN$, and arc $NL$ are in the ratio 5:3:4, then their measures are a multiple, $x$, of these numbers. So, $m(\text{arc } LM) = 5x$, $m(\text{arc } MN) = 3x$, and $m(\text{arc } MN) = 4x$. The arcs are adjacent and form the entire circle, so their sum is 360.

\[
5x + 3x + 4x = 360 \quad \text{Sum is } 360
\]

\[
12x = 360 \quad \text{Simplify}
\]

\[
x = 30 \quad \text{Divide each side by } 12.
\]

Therefore, $m(\text{arc } LM) = 5(30)$ or 150, $m(\text{arc } MN) = 3(30)$ or 90, and $m(\text{arc } NL) = 4(30)$ or 120.

**ANSWER:**
$m\overrightarrow{LM} = 150$, $m\overrightarrow{MN} = 90$, $m\overrightarrow{NL} = 120$
60. **OPEN ENDED** Draw a circle and locate three points on the circle. Estimate the measures of the three nonoverlapping arcs that are formed. Then use a protractor to find the measure of each arc. Label your circle with the arc measures.

**SOLUTION:**
Sample answer:

![Diagram of circle with central angles labeled: 127°, 131°, 102°.]

**ANSWER:**
Sample answer:

![Diagram of circle with central angles labeled: 127°, 131°, 102°.]

61. **CHALLENGE** The time shown on an analog clock is 8:10. What is the measure of the angle formed by the hands of the clock?

**SOLUTION:**
At 8:00, the minute hand of the clock will point at 12 and the hour hand at 8. At 8:10, the minute hand will point at 2 and the hour hand will have moved $\frac{10}{60} = \frac{1}{6}$ of the way between 8 and 9.

The sum of the measures of the central angles of a circle with no interior points in common is 360. The numbers on an analog clock divide it into 12 equal arcs and the central angle related to each arc between consecutive numbers has a measure of $\frac{360}{12} = 30$.

To find the measure of the angle formed by the hands, find the sum of the angles each hand makes with 12.

At 8:10, the minute hand is at 2 which is two arcs of 30 from 12 or a measure of 60.

When the hour hand is on 8, the angle between the hour hand and 12 is equal to four arcs of 30 or 120.

At 8:10, the hour hand has moved $\frac{1}{6}$ of the way from 8 to 9. Since the arc between 8 and 9 measures 30, the angle between the hand and 8 is $\frac{1}{6} (30) = 5$. The angle between the hour hand and 12 is 120 - 5 or 115.

The sum of the measures of the angles between the hands and 12 is 60 + 115 or 175.

Therefore, the measure of the angle formed by the hands of the clock at 8:10 is 175.

**ANSWER:**
175
62. **WRITING IN MATH** Describe the three different types of arcs in a circle and the method for finding the measure of each one.

**SOLUTION:**
Sample answer: Minor arc, major arc, semicircle; the measure of a minor arc equals the measure of the corresponding central angle. The measure of a major arc equals 360 minus the measure of the minor arc with the same endpoints. The measure of a semicircle is 180 since it is an arc with endpoints that lie on a diameter.

**ANSWER:**
Sample answer: Minor arc, major arc, semicircle; the measure of a minor arc equals the measure of the corresponding central angle. The measure of a major arc equals 360 minus the measure of the minor arc with the same endpoints. The measure of a semicircle is 180.

63. What is the value of $x$?

64. **SHORT RESPONSE** In $\odot B$, $m \angle BLM = 3x$ and $m \angle LBQ = 4x + 61$. What is the measure of $\angle PBQ$?

**SOLUTION:**
Here, $\angle BLM$ and $\angle LBQ$ form a linear pair. So, the sum of their measures is 180.

\[
3x + 4x + 61 = 180 \\
7x = 119 \\
x = 17
\]

$m \angle PBQ = 3(17)$ or 51

Since $\angle BLM$ and $\angle PBQ$ are vertical angles, $m \angle BLM = m \angle PBQ$.

Therefore, the measure of $\angle BLM$ is 51.

**ANSWER:**
51

65. **ALGEBRA** A rectangle’s width is represented by $x$ and its length by $y$. Which expression best represents the area of the rectangle if the length and width are tripled?

\[
F \ 3xy \\
G \ (xy)^2 \\
H \ 9xy \\
J \ (xy)^3
\]

**SOLUTION:**
The formula for the area of a rectangle is $A = \ell w$. The area of a rectangle with a length of $y$ and a width of $x$ is $A = xy$. When the dimensions are tripled, the length becomes $3y$ and width becomes $3x$. The area of the new rectangle is given by $A = (3x)(3y)$ or $9xy$.

Therefore, the correct choice is $H$.

**ANSWER:**
H
66. SAT/ACT What is the area of the shaded region if \( r = 4 \)?

\[ \text{A square} = s^2 \]
\[ \text{A square} = 8^2 \text{ or } 64 \]
\[ \text{A quarter circle} = \frac{\pi (4)^2}{4} \text{ or } 4\pi \]

Subtract the area of the four quarter circle sections from the square to find the area of the shaded region.
\[ A_{\text{shaded}} = 64 - 4(4\pi) \]
\[ A_{\text{shaded}} = 64 - 16\pi \]
Therefore, the correct choice is A.

**ANSWER:**
A

---

67. Name the center of the circle.

**SOLUTION:**
Since the circle is named circle \( J \), it has a center at \( J \).

**ANSWER:**
\( J \)

68. Identify a chord that is also a diameter.

**SOLUTION:**
A diameter of a circle is a chord that passes through the center and is made up of collinear radii. \( LN \) passes through the center, so it is a diameter.

**ANSWER:**
\( LN \)

69. If \( LN = 12.4 \), what is \( JM \)?

**SOLUTION:**
Here, \( JM \) is a radius and \( LN \) is a diameter. The radius is half the diameter. Therefore,
\[ JM = \frac{LN}{2} = 6.2 \text{ units.} \]

**ANSWER:**
6.2
**10-2 Measuring Angles and Arcs**

Graph the image of each polygon with the given vertices after a dilation centered at the origin with the given scale factor.

70. $X(-1, 2)$, $Y(2, 1)$, $Z(-1, -2)$; $r = 3$

**SOLUTION:**
Multiply the $x$- and $y$-coordinates of each vertex by the scale factor $k$. That is, $(x, y) \rightarrow (kx, ky)$.
Here multiply the $x$- and $y$-coordinates by the scale factor 3.

$X(-1, 2) \rightarrow X'(3, 6)$
$Y(2, 1) \rightarrow Y'(6, 3)$
$Z(-1, -2) \rightarrow Z'(-3, -6)$

**ANSWER:**

71. $A(-4, 4)$, $B(4, 4)$, $C(4, -4)$, $D(-4, -4)$; $r = 0.25$

**SOLUTION:**
Multiply the $x$- and $y$-coordinates of each vertex by the scale factor $k$. That is, $(x, y) \rightarrow (kx, ky)$.
Here multiply the $x$- and $y$-coordinates by the scale factor 0.25.

$A(-4, 4) \rightarrow A'(1, 1)$
$B(4, 4) \rightarrow B'(1, 1)$
$C(4, -4) \rightarrow C'(1, -1)$
$D(-4, -4) \rightarrow D'(-1, -1)$

**ANSWER:**
72. **BASEBALL** The diagram shows some dimensions of Comiskey Park in Chicago, Illinois. \(BD\) is a segment from home plate to dead center field, and \(AE\) is a segment from the left field foul pole to the right field foul pole. If the center fielder is standing at \(C\), how far is he from home plate?

**SOLUTION:**
Here, \(\triangle ABC\) is a 45°- 45°- 90° triangle and the length of leg \(BC\) is the distance from the center fielder to home plate. Use Theorem 8.8 to find \(x\).

\[
h = t\sqrt{2} \quad \text{Theorem 8.8}
\]

\[
347 = x\sqrt{2} \quad \text{Substitution}
\]

\[
\frac{347}{\sqrt{2}} = x \quad \text{Divide each side by} \sqrt{2}.
\]

\[
x = \frac{347\sqrt{2}}{2} \quad \text{Rationalize the denominator}
\]

Therefore, the center fielder is standing \(\frac{347\sqrt{2}}{2}\) or about 245.4 feet away from home plate.

**ANSWER:**
\[
\frac{347\sqrt{2}}{2} \approx 245.4 \text{ ft}
\]
10-2 Measuring Angles and Arcs

SOLUTION:
Since 36 is the measure of the altitude drawn to the hypotenuse of the large right triangle, 36 is the geometric mean of the lengths of the two segments that make up the hypotenuse, 6x and x.

\[ \frac{6x}{36} = \frac{2x}{x} \]

Geometric Mean (Altiude) Theorem

\[ 6x = 1296 \]

Cross Products are equal.

\[ x^2 = 216 \]

Divide each side by 6.

\[ x = \sqrt{6} \text{ or about 2.449} \]

Use a calculator to simplify.

The length of the hypotenuse is \( x + 6x \) or 7x. Since \( y \) is the measure of a leg of the right triangle, \( y \) is the geometric mean of \( x \) and \( 7x \).

\[ \frac{x}{y} = \frac{y}{7x} \]

Geometric Mean (Leg) Theorem

\[ \frac{\sqrt{6}}{y} = \frac{y}{7\sqrt{6}} \]

\[ y^2 = 1512 \]

Cross Products are equal.

\[ y = 6\sqrt{42} \text{ or about 38.9} \]

Use a calculator to simplify.

Since \( z \) is the measure of the other leg of the right triangle, \( z \) is the geometric mean of \( 6x \) and \( 7x \).

\[ \frac{5x}{z} = \frac{7x}{z} \]

Geometric Mean (Leg) Theorem

\[ \frac{5\sqrt{6}}{z} = \frac{7\sqrt{6}}{z} \]

Cross Products are equal.

\[ z^2 = 5040 \]

\[ z = 36\sqrt{7} \text{ or about 95.2} \]

Use a calculator to simplify.

Therefore, \( x = 6\sqrt{6} \text{ or about 14.7} \), \( y = 6\sqrt{42} \text{ or about 38.9} \), and \( z = 36\sqrt{7} \text{ or about 95.2} \).

ANSWER:
\[ x = 6\sqrt{6} \approx 14.7; y = 6\sqrt{42} \approx 38.9; z = 36\sqrt{7} \approx 95.2 \]

Find \( x \).

75. \( 24^2 + x^2 = 26^2 \)

SOLUTION:

\[ 24^2 + x^2 = 26^2 \] Given

\[ 576 + x^2 = 676 \]

Simplify

\[ x^2 = 100 \]

Subtract 576 from each side

\[ x = 10, -10 \]

Take the positive and negative square root of each side

ANSWER:
\[ 10, -10 \]